Measurements of All Complex Permeability Tensor Components and the Effective Line Widths of Microwave Ferrites Using Dielectric Ring Resonators

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Abstract —**A method of measuring all the complex permeability tensor components of microwave ferrites using a single cylindrical ferrite sample is described. Two dielectric ring resonators having the same height and internal diameter but different external diameters, operating on $HE^{\pm}_{111}$ and H_{011} modes respectively, are applied for these measurements. Permeability tensor components are computed from the measured resonant frequencies and Q factors of these resonators with and without the ferrite sample. Computations are based on the exact eigenvalue equations for these modes. Measurements of all permeability tensor components versus static magnetic field intensity, performed for different ferrite materials, generally confirm results obtained by earlier researchers but they also contain certain new aspects concerning relations between particular permeability tensor components below saturation.**

I. Introduction

PRACTICAL applications of microwave ferrites are inseparably connected with their unique magnetic properties at microwave frequencies. When a uniform static magnetic field is applied to a ferrite material along the z axis of a cylindrical coordinate system, the permeability tensor in this coordinate system takes the following well-known form:

$$\overleftrightarrow{\mu} = \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}. \quad (1)$$

All tensor components in a real medium are complex owing to the existence of magnetic losses. Particular tensor components depend on ferrite material properties, such as saturation magnetization, chemical composition, porosity, and grain size, and also on frequency and the static magnetic field strength applied to the ferrite material. Usually the real parts of permeability tensor components are assessed theoretically using two different sets of expressions. The first originates from well-known ferromagnetic resonance theory and it is valid when the ferrite material is saturated. The second set (valid below saturation) uses semiempirical formulas developed by Rado [1], Schlömann [2], Green and Sandy [3], and others. Only a few quantities are required to compute the real parts of the permeability tensor from the above-mentioned formulas: saturation magnetization, static magnetic field strength or average magnetization (below saturation), demagnetizing factor, frequency, and resonance line width, ΔH. These theoretically computed tensor components are in very good agreement with experiment, especially for the static magnetic field intensities sufficient for saturation of the ferrite material. For average magnetization greater than the remanence value but for small internal field intensities, the mentioned expressions are rather inexact. Hagelin [4] and Filipsson and Hansson [5] developed formulas allowing the computation of the real parts of the permeability tensor in these intermediate regions. Additional information about hysteresis parameters is required to use their formulas.

The imaginary parts of permeability tensor components generally cannot be predicted on the basis of knowledge of static or low-frequency parameters. Loss parameters should be measured at microwave frequencies. Near ferromagnetic resonance losses can be expressed by the resonance line width, ΔH. For partially magnetized material another loss parameter is introduced [6], [7] called, by analogy to the previous one, the effective line width, ΔH_{eff}. Although it is not generally possible to calculate components independently of all the imaginary parts of the permeability tensor from the value of ΔH_{eff}, it is a useful parameter. It is recommended by IEC [8] that it be measured as the only parameter characterizing nonresonance losses at low microwave power levels. All theories which make it possible to compute magnetic properties of ferrite materials at microwave frequencies are ultimately subject to experimental verification by microwave measurements of permeability tensor components appearing in (1) versus the static magnetic field magnetizing the ferrite. The most accurate methods of measuring these parameters at microwave frequencies are resonance methods utilizing resonant cavities containing ferrite samples [9]–[14] or a ferrite sample as a resonator [15], [16]. Generally, three independent measurements are required to find the three permeability tensor components at a

Manuscript received May 10, 1990; revised March 13, 1991.
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IEEE Log Number 9100139.

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certain fixed static magnetic field value. Theoretically, these independent measurements would be performed on different resonant systems, on different resonant modes, or for different static magnetic field directions. In practice only a few particular measuring systems are used. The most commonly used system is a cylindrical cavity containing a cylindrical sample of full height operating on two circularly polarized oppositely rotating TM^{+}_{110} and TM^{-}_{110} modes [9]–[13]. Measurements of the resonant frequencies and unloaded Q factors of these modes, with and without a ferrite sample, make it possible to compute two permeability tensor components, μ and $\varkappa$ (assuming that the complex permittivity of the ferrite material is known from independent measurements). Computations of the above parameters on the basis of experimental data were usually performed with the aid of a cavity perturbation method by earlier researchers [9], [10], [15] or, more recently [12], [13], [16], by a rigorous solution to the wave equation. The parallel component μ_z is usually determined on the basis of independent experiments. The most frequently used method for measuring this component [3], [10] employs small spherical samples placed at the center of a rectangular cavity operating in the TE_{102} mode. The perturbation method is used for computations of μ_z, since in this case the most recent exact solutions are not available.

In this paper an alternative method is presented for determining the real and imaginary parts of all three permeability tensor components using only one cylindrical ferrite sample. The method is based on the simultaneous utilization of the $HE^{\pm}_{111}$ and H_{011} modes, which occur in dielectric ring resonators containing the cylindrical ferrite sample. Employing the corresponding eigenvalue equation for these modes, the material parameters are computed from the measured resonant frequencies and Q factors of two dielectric ring resonators with and without the ferrite sample. To minimize frequency variations in the measuring system, two dielectric resonators are used having the same height and internal diameter but different external diameters. The external diameters are chosen so as to obtain similar resonant frequencies of the HE_{111} mode (the resonator having smaller diameter) and of the H_{011} mode (the resonator having greater diameter) for resonators containing typical nonmagnetized ferrite sample. The resonators are coupled to the external circuit through four identical loop-terminated coaxial cables, as shown in Fig. 1. The air gap which always exists between the ferrite sample and the dielectric resonator is rigorously taken into account in the eigenvalue equation.

II. Theory

Suppose initially that the conductivity of the two parallel metal plates and their radii in Fig. 1 are infinitely large. In this case it is possible to find exact expressions describing the electromagnetic fields in the ferrite rod and the dielectric cylindrical layers. Corresponding formulas for the electromagnetic fields in a ferrite rod and in

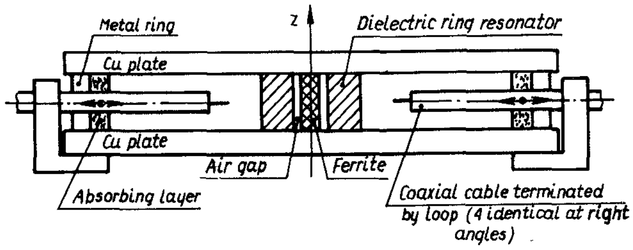


Fig. 1. Schematic diagram of the resonant systems used for measurements of permeability tensor components of a cylindrical ferrite sample.

dielectric cylindrical layers can be found in several papers and textbooks (e.g. [17]–[22]) and they will not be reproduced here. At the boundaries between particular regions the electromagnetic fields must satisfy well-known boundary conditions, which create a system of linear equations with respect to the constant coefficients appearing in the field expressions. The system of equations has nontrivial solutions if the corresponding determinant of the system vanishes. For fixed values of dimensions and material properties of the resonant system it is possible to find eigenfrequencies as roots of the nonlinear (with respect to frequency) equation in the form of a determinant. However, our task is to find material properties from measurements of resonant frequencies and unloaded Q factors of the dielectric ring resonators with and without the ferrite sample. We restrict our consideration to low-loss ferrites, assuming that the imaginary parts of all permeability tensor components and the imaginary parts of ferrite permittivity are small and so do not affect the resonant frequencies. When this is true, the real parts of the permeability tensor components and permittivities can be found from measured resonant frequency values in the following manner. As a first step, the real parts of the permittivities of the dielectric ring resonators are found (independently for each resonator) from measurements of the resonant frequencies and the geometrical dimensions of the empty resonators operating in the HE_{111} and H_{011} modes respectively. The real parts of the permittivities, $\mathrm{Re}(\epsilon_r)$, are computed using simplified (no ferrite) eigenvalue equations for these resonators in determinant form. As a second step, the permittivity of the ferrite sample, $\mathrm{Re}(\epsilon_f)$, and its scalar permeability, $\mathrm{Re}(\mu_d)$, are computed from measurements of the resonant frequencies of these resonators containing a completely demagnetized ferrite sample operating in the HE_{111} and H_{011} modes respectively. These parameters are computed as the solutions to the following system of nonlinear (with respect to $\mathrm{Re}(\epsilon_f)$ and $\mathrm{Re}(\mu_d)$) equations:

$$F_1\left(\mathrm{Re}(\epsilon_f), \mathrm{Re}(\mu_d), f^1_{r0}\right) = 0 \tag{2}$$

$$F_2\left(\mathrm{Re}(\epsilon_f), \mathrm{Re}(\mu_d), f^0_{r0}\right) = 0 \tag{3}$$

where f^1_{r0} and f^0_{r0} are measured resonant frequencies of the HE_{111} and H_{011} modes respectively for resonators containing a completely demagnetized ferrite sample. Functions F_1 and F_2 are eigenvalue equations in determinant form for proper modes of these resonators, assuming permeability to be a scalar denoted by μ_d. As the last step for each required value of the static magnetic field magnetizing the ferrite sample, the real parts of permeability tensor components are found as the solutions to the following system of nonlinear (with respect to the real parts of permeability tensor components) equations:

$$F_3(\mathrm{Re}(\mu), \mathrm{Re}(\varkappa), \mathrm{Re}(\mu_z), f_r^-) = 0 \quad (4)$$

$$F_4(\mathrm{Re}(\mu), \mathrm{Re}(\varkappa), \mathrm{Re}(\mu_z), f_r^+) = 0 \quad (5)$$

$$F_5(\mathrm{Re}(\mu), \mathrm{Re}(\varkappa), \mathrm{Re}(\mu_z), f_r^0) = 0 \quad (6)$$

where f_r^-, f_r^+, and f_r^0 are measured resonant frequencies of the HE_{111}^-, HE_{111}^+, and H_{011} modes respectively for resonators containing magnetized ferrite sample at a fixed static external magnetic field intensity. The first two frequencies are measured for the resonator having the smaller external diameter, while the third is for the resonator having the greater external diameter. As in the previous case, functions F_3, F_4, and F_5 are eigenvalue equations in determinant form for proper modes of the corresponding resonators. Theoretically any optimization procedure can be used to solve the systems of equations (2) and (3) and (4)–(6). I have used the well-known Newton iteration method to solve them. The convergence of this method is rapid, and a relative accuracy of about 10^{-4} for each unknown $\mathrm{Re}(\mu)$, $\mathrm{Re}(\varkappa)$, and $\mathrm{Re}(\mu_z)$ is obtained after a few iterations.

The imaginary parts of the permeability tensor components can be found after computing their real parts as the solutions to the following system of linear (with respect to the imaginary parts of permeability tensor components) equations:

$$\begin{aligned}(Q^-)^{-1} = (Q_c^-)^{-1} &+ p_{\epsilon r}^- \mathrm{Im}(\epsilon_r)/\mathrm{Re}(\epsilon_r) \\ &+ p_{\epsilon f}^- \mathrm{Im}(\epsilon_f)/\mathrm{Re}(\epsilon_f) \\ &+ p_{\mu}^- \mathrm{Im}(\mu)/\mathrm{Re}(\mu) + p_{\varkappa}^- \mathrm{Im}(\varkappa)/\mathrm{Re}(\varkappa) \\ &+ p_{\mu_z}^- \mathrm{Im}(\mu_z)/\mathrm{Re}(\mu_z)\end{aligned} \quad (7)$$

$$\begin{aligned}(Q^+)^{-1} = (Q_c^+)^{-1} &+ p_{\epsilon r}^+ \mathrm{Im}(\epsilon_r)/\mathrm{Re}(\epsilon_r) \\ &+ p_{\epsilon f}^+ \mathrm{Im}(\epsilon_f)/\mathrm{Re}(\epsilon_f) \\ &+ p_{\mu}^+ \mathrm{Im}(\mu)/\mathrm{Re}(\mu) + p_{\varkappa}^+ \mathrm{Im}(\varkappa)/\mathrm{Re}(\varkappa) \\ &+ p_{\mu_z}^+ \mathrm{Im}(\mu_z)/\mathrm{Re}(\mu_z)\end{aligned} \quad (8)$$

$$\begin{aligned}(Q^0)^{-1} = (Q_c^0)^{-1} &+ p_{\epsilon r}^0 \mathrm{Im}(\epsilon_r)/\mathrm{Re}(\epsilon_r) \\ &+ p_{\epsilon f}^0 \mathrm{Im}(\epsilon_f)/\mathrm{Re}(\epsilon_f) \\ &+ p_{\mu}^0 \mathrm{Im}(\mu)/\mathrm{Re}(\mu) + p_{\varkappa}^0 \mathrm{Im}(\varkappa)/\mathrm{Re}(\varkappa) \\ &+ p_{\mu_z}^0 \mathrm{Im}(\mu_z)/\mathrm{Re}(\mu_z).\end{aligned} \quad (9)$$

Here Q^-, Q^+, and Q^0 are the unloaded Q factors for the HE_{111}^-, HE_{111}^+, and H_{011} modes, respectively; Q_c^-, Q_c^+, and Q_c^0 are the Q factors depending on conductor losses in metal plates for the HE_{111}^-, HE_{111}^+, and H_{011} modes respectively; and p_x^-, p_x^+, and p_x^0 are the electric or magnetic energy filling factors computed by the perturbation method (not to be confused with the cavity perturbation method) as follows [16]:

$$p_x^+ = 2|\partial f_r^+/\partial x|x/f_r^+ \qquad p_x^- = 2|\partial f_r^-/\partial x|x/f_r^-$$

$$p_x^0 = 2|\partial f_r^0/\partial x|x/f_r^0$$

where x denotes $\mathrm{Re}(\epsilon_r)$, $\mathrm{Re}(\epsilon_f)$, $\mathrm{Re}(\mu)$, $\mathrm{Re}(\varkappa)$, or $\mathrm{Re}(\mu_z)$ respectively. The differentials appearing in the above expressions were computed numerically on the basis of the corresponding eigenvalue equations.

Equations (7)–(9) contain more than three unknowns. With the exception of the three imaginary parts of the permeability tensor, the values of Q_c^-, Q_c^+, Q_c^0, $\mathrm{Im}(\epsilon_r)$, and $\mathrm{Im}(\epsilon_f)$ should be deduced or measured in the following manner:

1) Imaginary parts of ϵ_r for dielectric ring resonators were measured at a fixed frequency by the method described in [23]. Values of $\mathrm{Im}(\epsilon_r)$ for each frequency value f_r^-, f_r^+, and f_r^0 were calculated assuming a linear increase of $\mathrm{Im}(\epsilon_r)$ with frequency.
2) Q_c factors have been deduced on the basis of measurements of the unloaded Q factors of the corresponding empty dielectric ring resonators and previously measured values of $\mathrm{Im}(\epsilon_r)$. Then it has been assumed that Q_c factors for resonators with ferrite sample are essentially the same as for empty resonators (corrections to their values being made only as a consequence of the change of the surface resistance value with frequency).
3) The imaginary part of permittivity, $\mathrm{Im}(\epsilon_f)$, of the ferrite sample has been found together with the imaginary part of its scalar permeability, $\mathrm{Im}(\mu_d)$, on the basis of measurements of the unloaded Q factors for resonators containing a completely demagnetized sample. In this case (8) and (9) becomes identical and include only one magnetic loss parameter, $\mathrm{Im}(\mu_d)$.

The frequency dependence of all permeability tensor components and of the scalar permeability μ_d has been taken into account in the computations. Frequency corrections were different for the real and imaginary parts of permeability tensor components and they are based on formulas presented in [3] and [24] expressing the dependence of particular tensor components on frequency. Frequency corrections of the real parts of the permeability tensor components were made employing two different sets of formulas: one below saturation and the other above saturation. Only one parameter was used to make corrections to the real parts of permeability tensor components, namely the magnetization saturation, M_s. Average magnetization or the relative internal field value (and also the proper set of formulas) were deduced from the values of $\mathrm{Re}(\mu)$ and $\mathrm{Re}(\varkappa)$ obtained during the first Newton iteration in computations of the real parts of permeability tensor components (the first iteration being performed without frequency corrections).

The imaginary parts of the tensor components were corrected assuming another parameter—the index N of

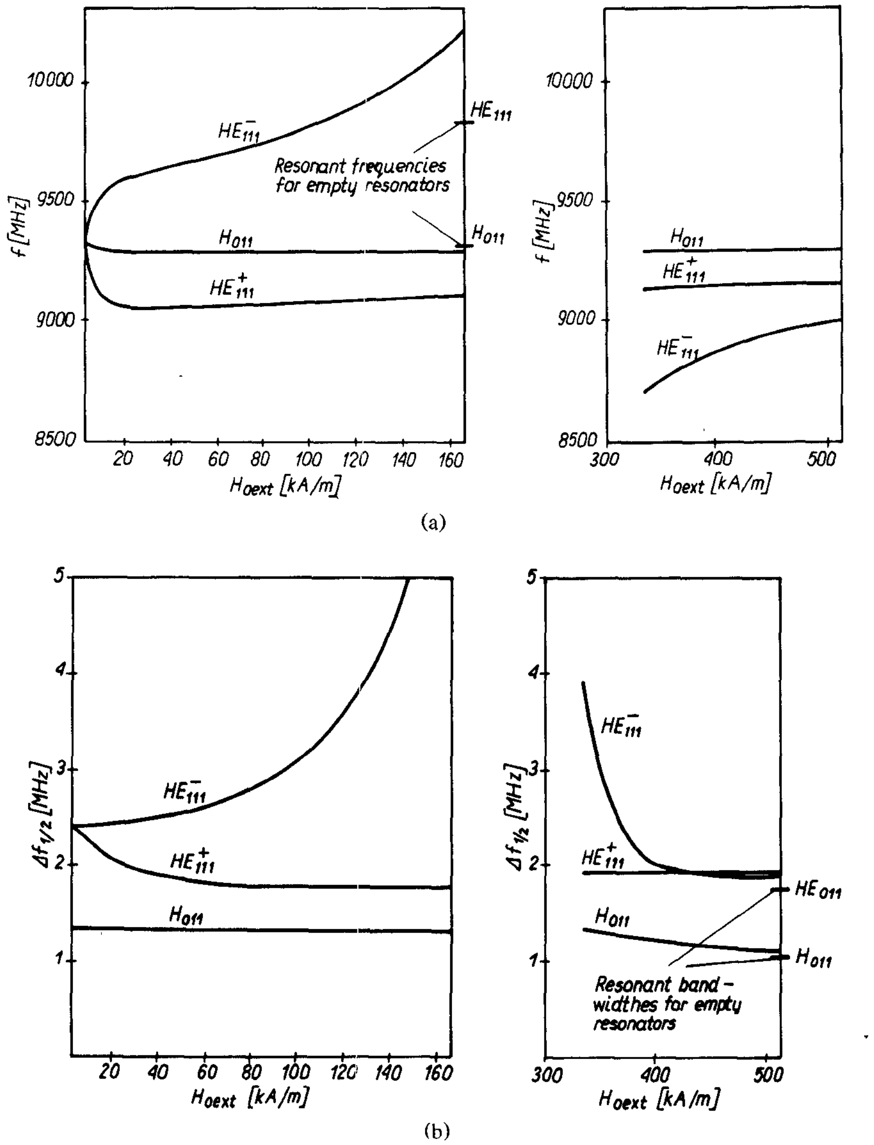


Fig. 2. Results of measurements on the dielectric ring resonators containing yttrium garnet sample versus static external magnetic field intensity (the field is applied along resonators axis): (a) resonant frequencies; (b) resonant frequency bandwidths.

the power describing the frequency dependence of the imaginary parts of permeability tensor components. It has been assumed that all imaginary parts of the permeability tensor and the imaginary part of the scalar permeability change with frequency according to the simple power law equation [24]

$$\mathrm{Im}[\mu_d(f)] = \mathrm{Im}[\mu_d(f_0)] * (f_0/f)^N.$$

Usually frequency corrections for both the real and the imaginary parts of the permeability tensor are not very significant since in practice the resonant frequencies for the $HE_{111}^{\pm}$ modes rarely differ more than 10% from the resonant frequency of the H_{011} mode. Theoretically, they would be much greater near the ferromagnetic resonance but in such cases measurements on relatively large samples cannot be performed because of high absorption.

All computations described in this section were organized in the form of a user-friendly computer program (available from the author upon request). Computations of all permeability tensor components with the aid of this program take about 1 min for one static magnetic field value on AMSTRAD 1512 PC.

III. Experiments

Experiments were performed in X band using hot-pressed alumina resonators having heights of 8.40 mm, internal diameters of 2.01 mm, external diameters of 8.40 mm and 11.62 mm respectively, and $\mathrm{Re}(\epsilon_r) \approx 10.0$. (The exact values of $\mathrm{Re}(\epsilon_r)$ were computed for each resonator and each experiment from the measured resonant frequencies of the empty resonators.) The unloaded Q factors of the empty resonators situated between polished copper plates were 5600 for the HE_{111} mode and 9200 for the H_{011} mode (the dielectric loss tangent of the hot-pressed alumina resonators at 10 GHz being about

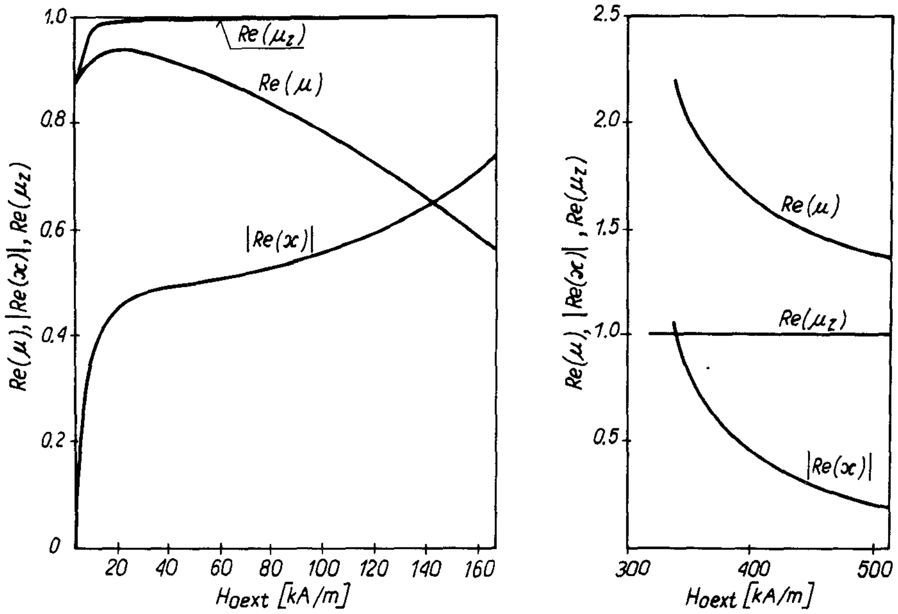


Fig. 3. Real parts of permeability tensor components of the yttrium garnet sample ($4\pi M_s = 1750$ G, $D_f = 2.00$ mm, $\mathrm{Re}(\epsilon_f) = 15.4$) versus static external magnetic field intensity. These results were computed from data shown in Fig. 2(a).

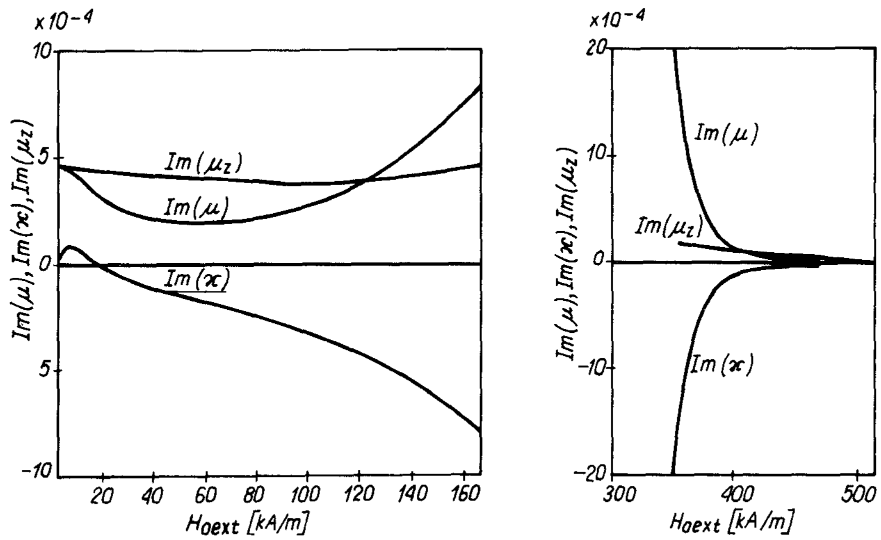


Fig. 4. Imaginary parts of permeability tensor components of the yttrium garnet sample ($4\pi M_s = 1750$ G, $D_f = 2.00$ mm, $\mathrm{Re}(\epsilon_f) = 15.4$) versus static external magnetic field intensity. The results were computed from data shown in Fig. 2.

3×10^{-5}). Measurements of the resonant frequencies and Q factors were performed using a Q factor meter made at the author's institute, allowing an accuracy in the Q-factor measurements of about 1% and an accuracy in the resonant frequency measurements to six or seven digits. Since it was not possible to obtain ferrite samples from other sources, all ferrite samples used in experiments were manufactured by POLFER (Poland).

Typical measurement data versus external static magnetic field intensity for yttrium iron garnet sample are presented in Fig. 2. The results of computations of the real and imaginary parts of permeability tensor components on the basis of these data are presented in Figs. 3 and 4. The results of measurements of permeability tensor components for other ferrite materials are presented in Figs. 5–10. Since the dimensions of the ferrite samples were relatively large (1.98–2.00 mm diameter and 8.40 mm height), measurements of the resonant frequencies and Q factors were not possible in the static field intensity region close to the resonance field values. The data plotted in Figs. 2–10 were obtained by starting from zero field for completely demagnetized samples. When the ferrite sample was measured again, in another resonator, it was first demagnetized.

The accuracy of the measurements of the scalar permeability and of the permeability tensor components depends primarily on errors in the measurement data in the H_{011} resonator. This can be explained as follows. The resonant frequency for the H_{011} mode depends predominantly on $\mathrm{Re}(\mu_d)$, while for the HE_{111} mode it depends

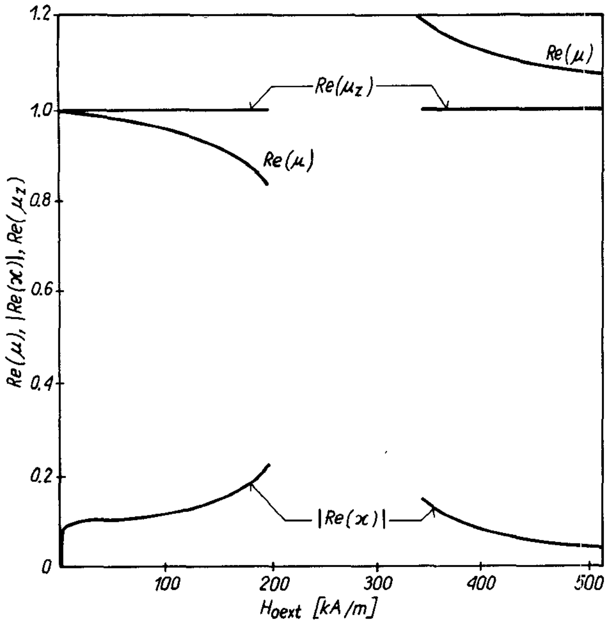


Fig. 5. Real parts of permeability tensor components of a substituted garnet sample ($4\pi M_s = 350$ G, $D_f = 1.98$ mm, $\mathrm{Re}(\epsilon_f) = 14.0$) versus static external magnetic field intensity.

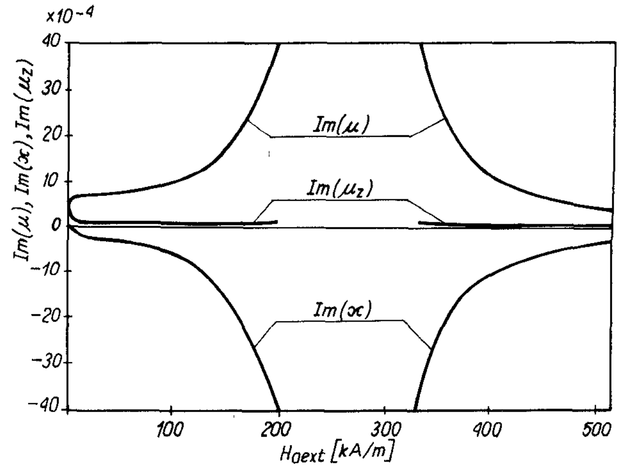


Fig. 6. Imaginary parts of permeability tensor components of a substituted garnet sample ($4\pi M_s = 350$ G, $D_f = 1.98$ mm, $\mathrm{Re}(\epsilon_f) = 14.0$) versus static external magnetic field intensity.

on $\mathrm{Re}(\mu_d)$ and $\mathrm{Re}(\epsilon_f)$. Therefore, solving the system of equations (2) and (3), one obtains the $\mathrm{Re}(\mu_d)$ value determined by measurement data of the H_{011} resonator and the $\mathrm{Re}(\epsilon_f)$ value determined by measurement data of both the H_{011} and HE_{111} resonators. In further measurements of the permeability tensor components the $\mathrm{Re}(\epsilon_f)$ value is assumed to be constant. The validity of the results obtained in measuring $\mathrm{Re}(\mu_d)$ may be additionally checked by measurements at sufficiently high external field intensities (for high field intensities the parallel component $\mathrm{Re}(\mu_z)$ should approach unity). In fact, for all materials measured, the values of $\mathrm{Re}(\mu_z)$ at $H_{0\mathrm{ext}} = 500$ kA/m never differed from unity by more than 0.15%.

Another test which confirms the high accuracy of the measurements of magnetic properties has been performed on dielectric samples. For these samples, the real parts of permeability computed from the measurement data again never differed from unity by more than 0.15%. However, it should be pointed out that the error of determining the real part of permittivity can be as great as a few percent in such cases when the height of the ferrite sample is smaller than that of the dielectric HE_{111} mode resonator or when the contact between the HE_{111} mode resonator and Cu plates is unrepeatable. To minimize the latter, the dielectric resonator holder has been constructed so as to make repeatability of the resonant frequency of the HE_{111} mode resonator better than 2 MHz (after its disassembly). It is worth noting that the accuracy of the off-diagonal permeability tensor component determination depends on the symmetry of the HE_{111} mode resonator. Because of unavoidable mechanical imperfections the resonant frequency of this mode is split into two frequencies, even for an empty resonator. For our resonator this initial split was smaller than 5 MHz, which caused an error in determining $\mathrm{Re}(\varkappa)$ of less than 0.005 (this value was assessed from Figs. 2 and 3).

In Fig. 11 results are presented of computations of the effective line widths, ΔH_{eff}, versus static magnetic field intensities for the materials whose permeability tensor components were measured earlier. The effective line widths were calculated from the following well-known formula [8], which can be expressed in terms of permeability tensor components and saturation magnetization, M_s, as follows:

$$\Delta H_{\mathrm{eff}} = 2M_s p/(p^2+q^2) \tag{10}$$

where

$$p = \mathrm{Im}(\mu) - \mathrm{Im}(\varkappa)$$

and

$$q = \mathrm{Re}(\mu) - \mathrm{Re}(\varkappa) - 1.$$

Results of all the experiments presented so far are to some extent different from those obtained by earlier researchers. The main differences and similarities can be stated as follows:

1) The real parts of the parallel component $\mathrm{Re}(\mu_z)$ below saturation are usually greater than the real parts of μ for a fixed average magnetization (equivalent in our case to a fixed static field value). These results are contrary to those obtained in [3], where it is postulated that $\mathrm{Re}(\mu_z)$ is smaller than $\mathrm{Re}(\mu)$ for a fixed average magnetization. The latter results given in [3] are in the form of the following relationships:

$$\mathrm{Re}(\mu) = \mathrm{Re}(\mu_d) + (1-\mathrm{Re}(\mu_d))(M/M_s)^{3/2} \tag{11}$$

$$\mathrm{Re}(\mu_z) = \mathrm{Re}(\mu_d)^{(1-(M/M_s)^{5/2})} \tag{12}$$

and they are commonly used for computations of the components $\mathrm{Re}(\mu)$ and $\mathrm{Re}(\mu_z)$ below saturation.

2) The values of the real part of μ in the low-field region never approach unity and its maximum value for the lithium–titanium ferrite can be as low as

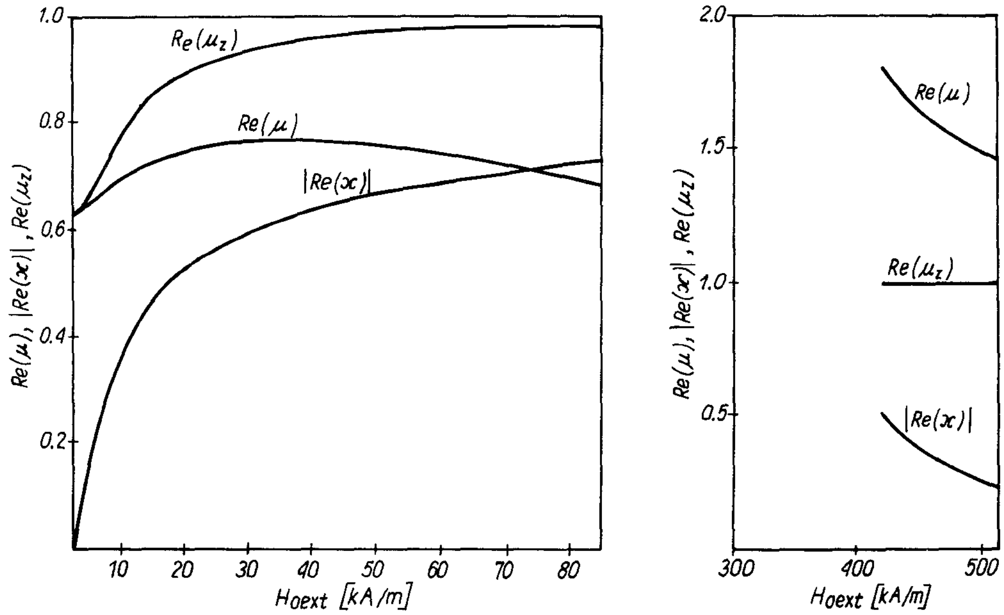


Fig. 7. Real parts of permeability tensor components of lithium–titanium ferrite sample ($4\pi M_s = 2600$ G, $D_f = 2.00$ mm, $\text{Re}(\epsilon_f) = 16.2$) versus static external magnetic field intensity.

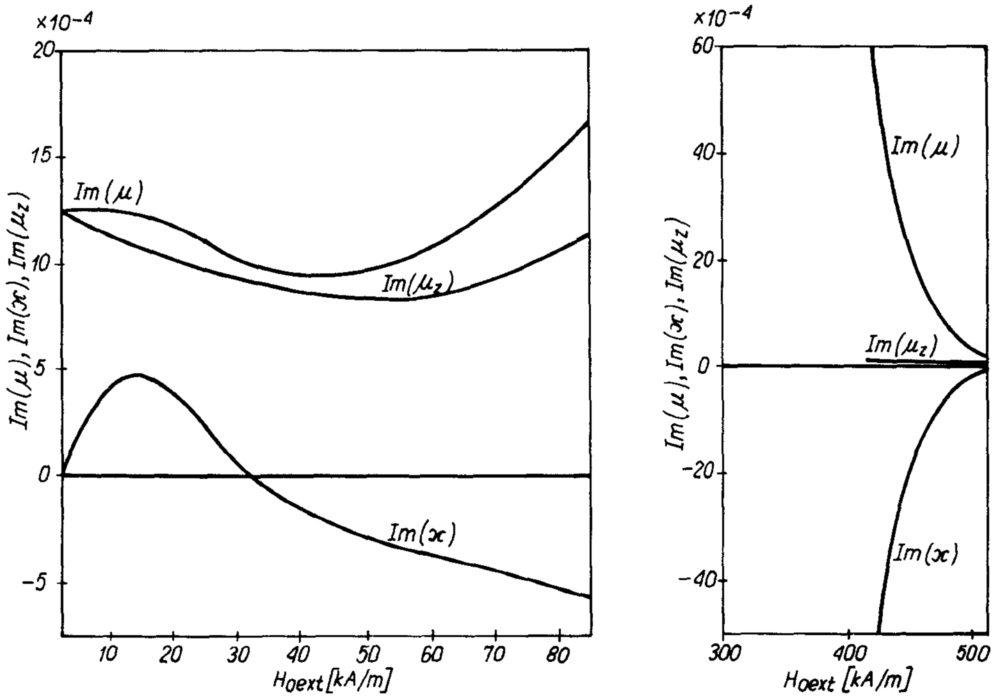


Fig. 8. Imaginary parts of permeability tensor components of the lithium–titanium ferrite sample ($4\pi M_s = 2600$ G, $D_f = 2.00$ mm, $\text{Re}(\epsilon_f) = 16.2$) versus static external magnetic field intensity.

0.77. Similar results were presented for other ferrite materials in [5] by Filipsson and Hansson.

3) The maximum value of $\text{Re}(\mu)$ in the low-field region and the value of the scalar permeability $\text{Re}(\mu_d)$ are usually smaller for ferrites having a greater saturation magnetization, but exceptions to this have been observed (see e.g. results for the lithium–titanium and the magnesium–manganese ferrites). Results obtained by Green and Sandy [3] also show such exceptions.

4) Relations between the real parts of the permeability tensor components for the high-field region correspond to the theoretical expression obtained from classical expressions. Similar results have been obtained by many researchers (e.g. [11]–[13]).

5) The imaginary parts of $\varkappa$ for ferrites having greater saturation magnetization for very low static field intensities have a sign that is opposite to that in the remaining region. Such phenomena have been observed by Gurevich [21].

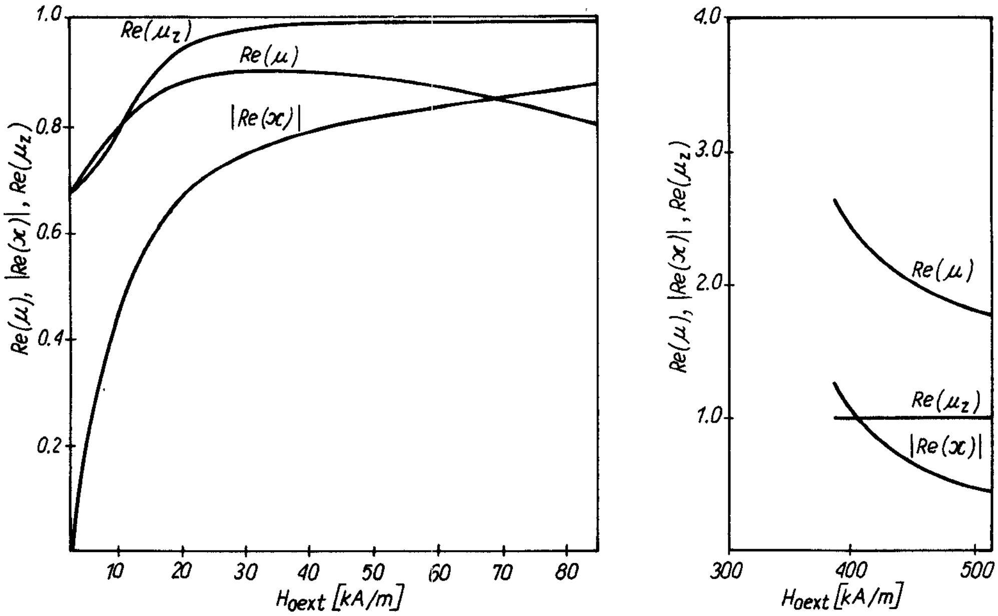


Fig. 9. Real parts of permeability tensor components of the magnesium–manganese ferrite sample ($4\pi M_s = 3000$ G, $D_f = 1.98$ mm, $\mathrm{Re}(\epsilon_f) = 12.2$) versus static external magnetic field intensity.

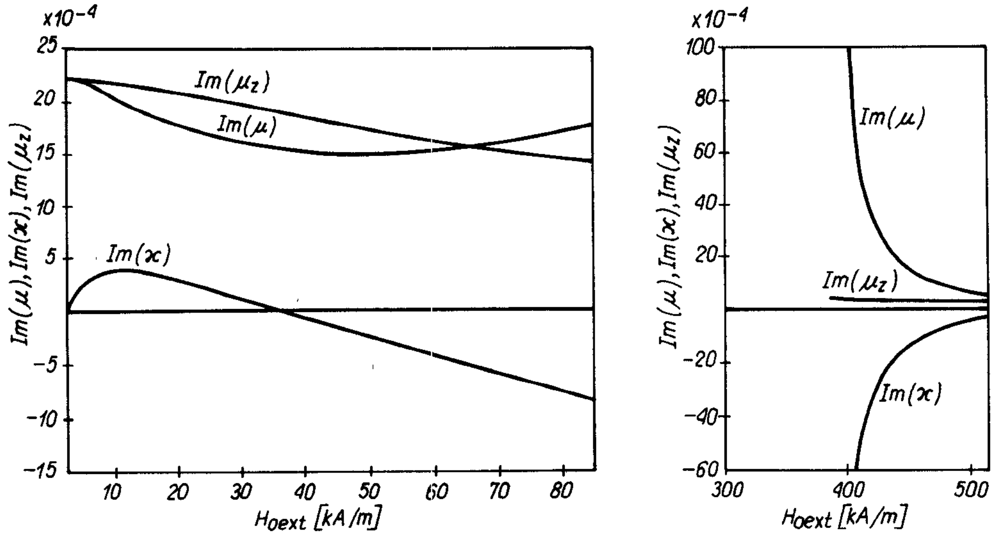


Fig. 10. Imaginary parts of permeability tensor components of the magnesium–manganese ferrite sample ($4\pi M_s = 3000$ G, $D_f = 1.98$ mm, $\mathrm{Re}(\epsilon_f) = 12.2$) versus static external magnetic field intensity.

6) The imaginary parts of the parallel component μ_z for materials having larger saturation magnetization values are high (close to $\mathrm{Im}(\mu_d)$ value) throughout the low-field region although the real parts of μ_z almost approach unity there. The external magnetic field value required to reduce these losses to an insignificant level, for such materials, is higher than the resonance field.

The validity of the method presented in this paper has been additionally confirmed by independent measurements of all permeability tensor components using two cylindrical ferrite resonators [16]. The resonators were made of the same yttrium garnet ($4\pi M_s = 1750$ G) as the sample in Figs. 2–4. Comparisons of the results were not possible at a fixed external static magnetic field intensity since the ferrite resonators had $L/d_f = 1$ while the samples used in the method described in this article had $L/d_f = 4.2$, so the demagnetization factors were also different. Therefore results were compared with respect to relations between particular tensor components of permeability tensor. Differences between the real parts of permeability tensor components obtained by those two methods were never greater than 1%, and between the imaginary parts they never exceeded 15%, so the validity of the new method was confirmed.

IV. Comparison of Different Methods

One of the most important aspects to be considered in choosing a resonant system is the accuracy that can be expected in measuring the imaginary parts of permeability

TABLE I
ELECTRIC AND MAGNETIC ENERGY FILLING FACTORS AND RELATIVE Q-FACTOR CHANGES CAUSED BY MAGNETIC LOSSES ($\mathrm{Im}(\mu_d) = 2\times10^{-4}$) FOR DIFFERENT CYLINDRICAL RESONANT SYSTEMS CONTAINING DEMAGNETIZED CYLINDRICAL FERRITE SAMPLES OF FULL HEIGHT HAVING $\mathrm{Re}(\epsilon_f)$ AND $\mathrm{Re}(\mu_d) = 1$

Resonant System	D_f (mm)	Q_0 (Typical)	p_{ϵ_f}	p_{μ_d}	$\Delta Q/Q$ [%]	Remarks
TM_{110} mode cavity	1.6	8000	0.0005	0.0053	0.82	D_f proposed by IEC [8]
	4.6	8000	0.0876	0.1067	17.0	
HE_{111} D.R.	2.0	5600	0.0798	0.1176	13.0	D.R. as in experiments
H_{011} D.R.	2.0	9200	0.0052	0.0723	13.3	D.R. as in experiments

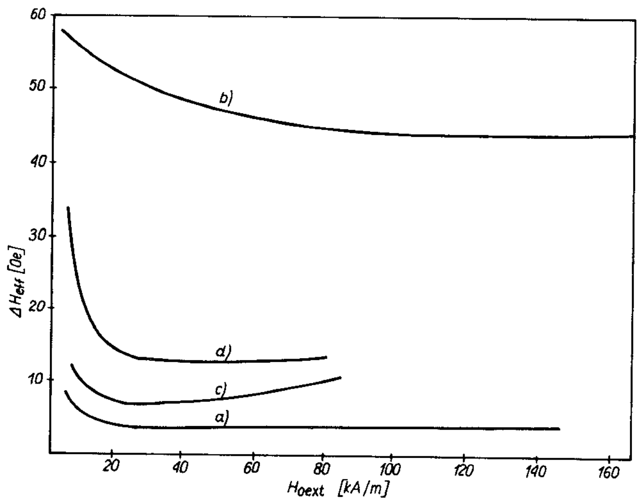


Fig. 11. Effective line widths versus static external magnetic field intensity computed for different materials on the basis of results shown in Figs. 3–10: *a*) yttrium iron garnet, *b*) substituted garnet, *c*) lithium–titanium ferrite, *d*) magnesium–manganese ferrite.

TABLE II
ELECTRIC AND MAGNETIC ENERGY FILLING FACTORS FOR DIELECTRIC RING $HE^{\pm}_{111}$ AND H_{011} MODE RESONATORS CONTAINING A MAGNETIZED FERRITE SAMPLE HAVING $\mathrm{Re}(\epsilon_f) = 13.6$, $\mathrm{Re}(\mu) = 0.859$, $\mathrm{Re}(\varkappa) = 0.785$, AND $D_f = 2.00$ mm

Resonant System	p_{ϵ_f}	p_μ	$p_\varkappa$	p_{μ_z}
HE^{+}_{111} D.R.	0.0764	0.0603	0.0560	0.0019
HE^{-}_{111} D.R.	0.0392	0.1920	−0.1640	0.0003
H_{011} D.R.	0.0045	0.0012	0.0002	0.0672

Parameters of the dielectric resonators are the same as in the experimental part of the paper.

tensor components. Let us consider this problem in detail for our method and for the methods employing the TM_{110} mode cavity. The minimum value of the magnetic losses which can be measured in any resonant system depends upon two factors: the sensitivity of the Q factor of the particular resonant system to magnetic losses introduced by the ferrite sample and the ability to measure small changes in the Q factor. The first of these factors depends on the unloaded Q factor of the resonant system without the ferrite sample and on the relative dimensions of the sample (more precisely, the magnetic energy filling factors defined in formulae (7)–(9)), while the second depends on the apparatus used for the Q-factor measurements. If all three imaginary tensor components are to be measured, attention should also be given to how particular imaginary parts of the permeability tensor components affect the Q factors of the particular resonant systems (or particular modes). This also can be expressed in terms of the magnetic energy filling factors p_μ, $p_\varkappa$, and p_{μ_z} appearing in formulae (7)–(9). In Table I results are given for the computations of the electric and magnetic energy filling factors for the HE_{111} and H_{011} mode resonators and for the TM_{110} cavity (40 mm diameter and 10 mm height) containing completely demagnetized ferrite samples. The last column of the table shows the relative change of the Q factor of the particular resonant system produced by ferrite losses: $\mathrm{Im}(\mu_d) = 2\times10^{-4}$. It is seen that the sample that is 2.0 mm in diameter causes Q-factor changes of the HE_{111} mode and the H_{011} mode dielectric resonators similar to those of the sample 4.6 mm in diameter in the case of the TM_{110} mode cavity. Influences of the imaginary parts of permittivity on the Q factors of the TM_{110} mode cavity and the HE_{111} mode D.R. expressed in terms of the electric energy filling factors p_{ϵ_f} are also similar in both cases. For the sample 1.6 mm in diameter (the value proposed in [8]) the relative change in the Q factor produced by the small magnetic losses ($\mathrm{Im}(\mu_d) = 2\times10^{-4}$) is smaller than 1% and requires sophisticated apparatus to be accurately measured.

Table II lists results of computations of the magnetic and electric energy filling factors p_μ, $p_\varkappa$, p_{μ_z}, and p_{ϵ_f} for the HE_{111} and H_{011} modes in the case of permeability tensor measurements. The parameters of the dielectric resonators have been assumed to be the same as in my experiments. It is seen that for the H_{011} mode the p_{μ_z} factor is much greater than the remaining coefficients and that for the two counterrotating $HE^{\pm}_{111}$ modes the p_μ factors have the same signs and the $p_\varkappa$ factors have opposite signs. This formally means that the system of equations (7)–(9) is well conditioned with respect to the unknowns $\mathrm{Im}(\mu)$, $\mathrm{Im}(\varkappa)$ and $\mathrm{Im}(\mu_z)$. Similar results were obtained for p_μ and $p_\varkappa$ in the case of a $TM^{\pm}_{110}$ mode cavity containing a sample 4.6 mm in diameter. It should be remarked that, for a rectangular TE_{102} mode cavity having the dimensions $40.0\times20.0\times10$ mm containing a ferrite sphere 2.0 mm in diameter, the p_{μ_z} factor is approximately equal to 0.002 (calculated from perturbation formula). Samples having greater diameters would produce greater p_{μ_z} values for such a cavity but perturbation theory is no longer valid for greater samples. Therefore, in the case of low-loss ferrites, the accuracy of measurements of the imaginary part of μ_z by means of this method is not too high.

V. Conclusions

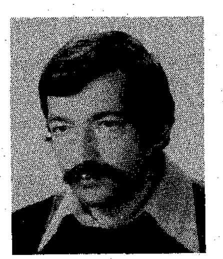
The method presented in this paper seems to be practical for measurements of permeability tensor components at frequencies below 10 GHz since it has the following advantages over existing cavity methods:

1) Only one ferrite sample is required for measurements of all tensor components.
2) Dimensions of a ferrite sample are smaller than in cavity methods (assuming the same magnetic energy filling factors in both cases). This would be important when measurements are to be performed at lower frequency bands (e.g. the S band).
3) Resonant systems used in experiments do not have insertion holes in metal plates, which affect measurements in cavity methods.
4) Assembling and disassembling the resonant systems does not change their Q factors since the dielectric resonators do not have unrepeatable metal-to-metal contacts.

For frequencies higher than 10 GHz cavity methods seem to be more practical because the dimensions of dielectric resonators become too small. A cylindrical H_{011} mode cavity would be used instead of an H_{011} mode dielectric resonator together with the TM_{110} cavity for measurements of all permeability tensor components on one cylindrical sample.

References

[1] G. T. Rado, "Theory of the microwave permeability tensor and Faraday effect in nonsaturated ferromagnetic materials," *Phys. Rev.*, vol. 89, p. 529, 1953.
[2] E. Schlömann, "Microwave behavior of partially magnetized ferrites," *J. Appl. Phys.*, vol. 41, pp. 204–214, Jan. 1970.
[3] J. J. Green and F. Sandy, "Microwave characterization of partially magnetized ferrites," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 641–645, June 1974.
[4] S. Hagelin, "Magnetization and effective internal magnetic field-strength in unsaturated ferrite disks," Res. Inst. of National Defence Report, C 30013-E2, Nov. 1974 (in Swedish).
[5] G. Filipsson and B. Hansson, "New theoretical models and computer programs for accurate design of microwave junction circulators," in *Proc. 7th European Microwave Conf.* (Copenhagen), 1977, pp. 637–641.
[6] Q. H. F. Vrehen, "Absorption and dispersion in porous and anisotropic polycrystalline ferrites at microwave frequencies," *J. Appl. Phys.*, vol. 40, pp. 1849–1860, Mar. 1969.
[7] C. E. Patton, "Effective linewidth due to porosity and anisotropy in policrystalline yttrium iron garnet and Ca-V-substituted yttrium iron garnet at 10 GHz," *Phys. Rev.*, vol. 179, pp. 352–358, Mar. 1969.
[8] *Measuring Methods for Properties of Gyromagnetic Materials Intended for Application at Microwave Frequencies*, IEC Standard, Publication 556, pp. 75–87, Geneva, 1982.
[9] R. C. LeCraw and E. G. Spencer, "Tensor permeabilities of ferrites below magnetic saturation," in *IRE Conv. Rec.* (New York), pt. 5, pp. 66–74, 1956.
[10] J. J. Green and T. Kohane, "Testing of ferrite materials for microwave applications," *Semicond. Prod. Solid State Technol.*, vol. 7, pp. 46–54, Aug. 1964.
[11] H. E. Bussey and L. A. Steinert, "Exact solution for a gyromagnetic sample and measurements on a ferrite," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 72–76, Jan. 1958.
[12] W. Müller-Gronau and I. Wolff, "A microwave method for the determination of the real parts of the magnetic and dielectric material parameters of premagnetized microwave ferrites," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 377–382, June 1983.
[13] W. Müller-Gronau, "Ein Messverfahren zur Bestimmung der komplexen dielektrischen und magnetischen Materialparameter von vormagnetisierten Mikrowellenferriten," Doktor-Ingenieurs Dissertation, Vom Fachbereich Elektrotechnik der Universität—Gesamthochschule—Duisburg zur Erlangung des akademischen Grades eines, Essen, 1984.
[14] P. Le Roux, F. Jecko, and G. Forterre, "Nouvelle methode de determination des parties reelles des parametres electriques et magnetiques des resonateurs a ferrites satures et non satures," *Annales des Telecommunications*, vol. 43, pp. 314–322, May/June 1988.
[15] N. Ogasawara, T. Fuse, T. Inui, and I. Saito, "Highly sensitive procedures for measuring permeabilities ($\mu_{\pm}$) for circularly polarized fields in microwave ferrites," *IEEE Trans. Magn.*, vol. MAG-12, pp. 256–259, May 1976.
[16] J. Krupka, "Resonant modes in shielded cylindrical ferrite and single crystal resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 691–697, Apr. 1989.
[17] M. L. Kales, "Modes in waveguides containing ferrites," *J. Appl. Phys.*, vol. 24, pp. 604–608, May 1953.
[18] R. A. Waldron, "Electromagnetic wave propagation in cylindrical waveguide containing gyromagnetic media," *J. Brit. Inst. Radio Eng.*, vol. 18, pp. 597–610, 671–690, 733–746, 1958.
[19] H. D. Godtmann and W. Haas, "Magnetodynamic modes in axially magnetized rods between two parallel conducting sheets," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 478–480, Aug. 1967.
[20] A. M. Duputz and A. C. Priou, "Computer analysis of microwave propagation in a ferrite loaded circular waveguide-optimization of phase-shifter longitudinal field section," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 601–613, June 1974.
[21] A. G. Gurevich, *Ferrites at Microwave Frequencies.* New York: Consultants Bureau Enterprises Inc., 1963, chs. 4 and 10.
[22] A. J. Baden Fuller, *Ferrites at Microwave Frequencies.* London: Peter Peregrinus, 1987, ch. 3.
[23] J. Krupka, "An accurate method of permittivity and loss tangent measurements of low loss dielectrics using $TE_{01\delta}$ dielectric resonators," in *Proc. Fifth Int. Conf. Dielectric Materials Measurements and Applications* (Canterbury, U.K.), June 27–29, 1988, pp. 322–325.
[24] J. J. Green and F. Sandy, "A catalog of low power loss parameters and high power thresholds for partially magnetized ferrites," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 645–651, June 1974.



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